INFLUENCE OF TURBULENT PRANDTL NUMBER ON HEAT TRANSFER OF A FLAT PLATE

A. Sh. Dorfman

UDC 536.24:532.526

In computations involving heat transfer in turbulent flow past bodies it is necessary to assume turbulent Prandtl number distribution across the boundary layer. A review and comparison of results obtained by different authors are given, e.g., in [1-5]. Unfortunately, the existing data are so contradictory that, at present, it does not appear to be possible to establish reliably a function that determines turbulent Prandtl number distribution across the boundary layer. The absence of sufficiently reliable and general results on the distribution of turbulent Prandtl number led to the result that in the majority of studies conducted in earlier years its value was assumed a constant and either close to or equal to one. The effect of turbulent Prandtl number on the intensity of heat transfer from a flat plate is numerically investigated in the present paper. The thermal, turbulent boundary layer equation is integrated for this purpose at different values of turbulent Prandtl number and results are compared with experimental data. Results from [6], where the thermal boundary layer was numerically integrated with $Pr_t = 1$ and compared with experimental data, were used for comparison in the present paper. The same numerical integration procedure as in [6] was used here.

The system of turbulent boundary layer equations is closed with the Mellor–Gibson model [7]. According to this model, the turbulent boundary layer is divided into three regions. In the inner region, including the laminar sublayer and the turbulent layer immediately close to it, the eddy viscosity ν_T is determined in the following form [7, 8] from similarity considerations,

$$1 + \frac{\nu_T}{\nu} = f\left(\frac{y^2}{\nu}\frac{\partial u}{\partial y}\right), \tag{1}$$

where ν is the kinematic viscosity coefficient; u and y are the streamwise velocity component and the transverse coordinate in the boundary layer. The form of the function f is established on the basis of known accurate measurements made by Laufer [7, 8].

In the outer region the eddy viscosity as in the majority of other modern simplest models is assumed to be independent of the transverse coordinate and is determined by the equation

$$\nu_T = 0.016 U\delta^*, \tag{2}$$

where U is the outer edge velocity; δ^* is the displacement thickness. In the midsection of the layer Prandtl's equation is used

$$\nu_t = \varkappa^2 y^2 |\partial u/\partial y|, \tag{3}$$

in which the mixing length is assumed to be proportional to the transverse coordinate ($\varkappa = 0.4$ is Karman's constant). The boundaries between regions are found by equating eddy viscosity from Eqs. (1) and (2) on the one hand and (3) on the other.

The above model for eddy viscosity was used in [7, 9] to compute velocity profiles in the inner and outer regions of the layer. The inner and outer profiles coincide over a certain segment in the intermediate region, completing the velocity profile in the turbulent boundary layer. The velocity and eddy viscosity distribution across the boundary layer in such a case is determined by functions of nondimensional coordinate $\eta = y/\Delta$ that depends on Reynold's number $Re_* = U\delta^*/\nu$ as a parameter ($\Delta = \delta^*/\sqrt{c_f/2}$, c_f is skin friction coefficient).

It was shown in [10] that under such conditions and a constant value of turbulent Prandtl number the thermal boundary layer equation can be comprehensively used; more details of the method are given in [11]. Characteristic features of computations at large and small values of Prandtl numbers associated with the need to consider the nature of damping of fluctuations in the viscous

Kiev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 81-85, July-August, 1984. Original article submitted June 9, 1983.

 0021-8944/84/2504-0572$08.50

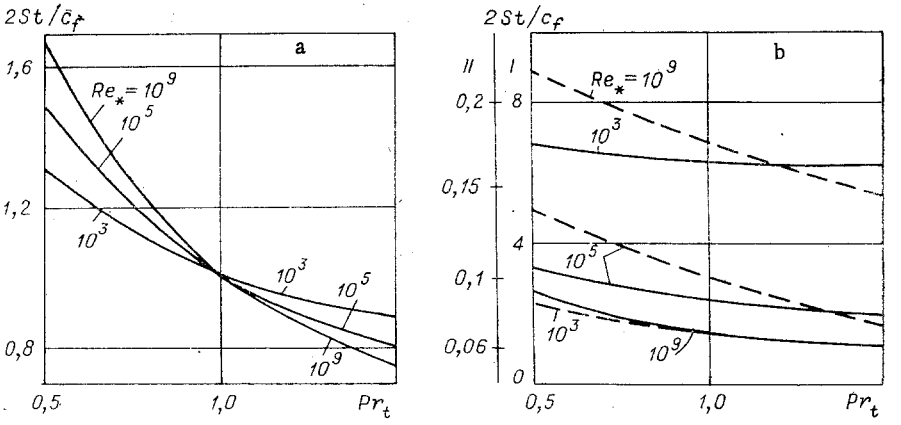

Fig. 1

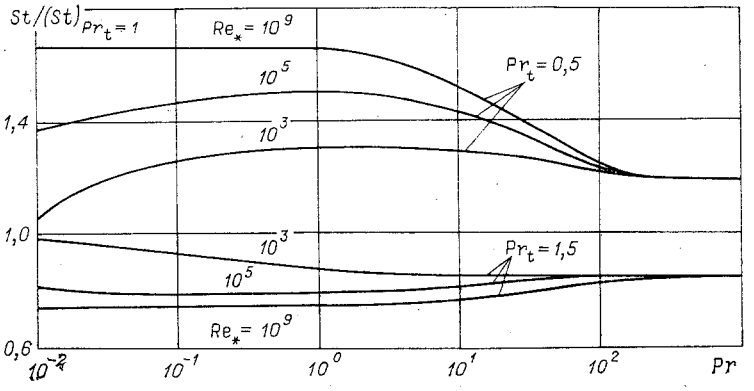

Fig. 2

sublayer in the first case and the behavior of eddy viscosity coefficient outside the dynamic layer in the second case are also discussed in [11].

Computations have been carried out for two values of turbulent Prandtl numbers Pr_t = 0.5 and 1,5, four values of Prandtl number Pr = 10^{-2}; 1; 10^2; 10^3; and three Reynolds numbers $Re_* = 10^3$ ($Re = Ux/\nu = 2.95\cdot10^5$); 10^5 ($7.93\cdot10^7$); 10^9 ($2.56\cdot10^{12}$). Computational results in the form of dependence of the coefficient of Reynolds analogy $2St/c_f$ on turbulent Prandtl number are shown in Fig. 1 (a: Pr = 1; b: Pr = 0.01 (continuous line, Scale I), Pr = 100 (dashed line, Scale II)). The corresponding data for Pr_t = 1 are taken from [6]. Computations for Pr_t = 1 showed [6] that at large Prandtl numbers the product $St\sqrt{2/c_f}Pr^{3/4}$ practically does not depend on Reynolds numbers and equals 0.113. The corresponding values for Pr_t = 0.5 and 1.5 are 0.136 and 0.096. In view of this, curves for $Pr = 10^3$ similar to those given in Fig. 1 are not plotted for other values of Prandtl numbers.

Computed results are given in Fig. 2 in the form of the ratio $St/(St)_{Pr_t=1}$ which determines the change in heat transfer with change in Pr_t, compared to heat transfer calculated for Pr_t = 1.

It follows from Fig. 2 that an increase in Pr_t leads to a reduction and its reduction to an increase in heat transfer compared to that at Pr_t = 1, and here an increase in Pr_t up to 1.5 is less appreciable than its reduction to 0.5; turbulent Prandtl number has the most appreciable effect when Prandtl number is close to unity, when Pr_t = 0.5 the difference in St from the corresponding value at Pr_t = 1 is a maximum when Pr = 1, $Re_* = 10^9$ and it is 67%, the corresponding difference at Pr_t = 1.5 is 25%; with an increase in Pr and a reduction in Re the effect of turbulent Prandtl number is reduced and when $Pr \geqslant 10^2$ it happens to be practically independent of Pr as well as Re. In this case the ratio $St/(St)_{Pr_t=1}$ at Pr = 0.5 becomes equal to $\sim$1.2, and when Pr_t = 1.5 it is $\sim$0.85. Comparatively small influence of Pr_t at large values of Prandtl numbers is explained by the presence of the major portion of the thin thermal layer in the viscous sublayer. With a decrease in Prandtl number the influence of Pr_t also decreases because of the increasing role of molecular heat conductivity.

In [6] it was shown that when Pr < 1 computational results from Pr_t = 1 are very well generalized by using Peclet number Pr = RePr as the independent variable. In this case

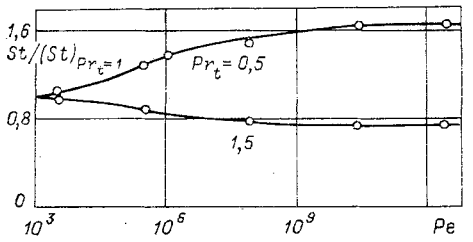

Fig. 3

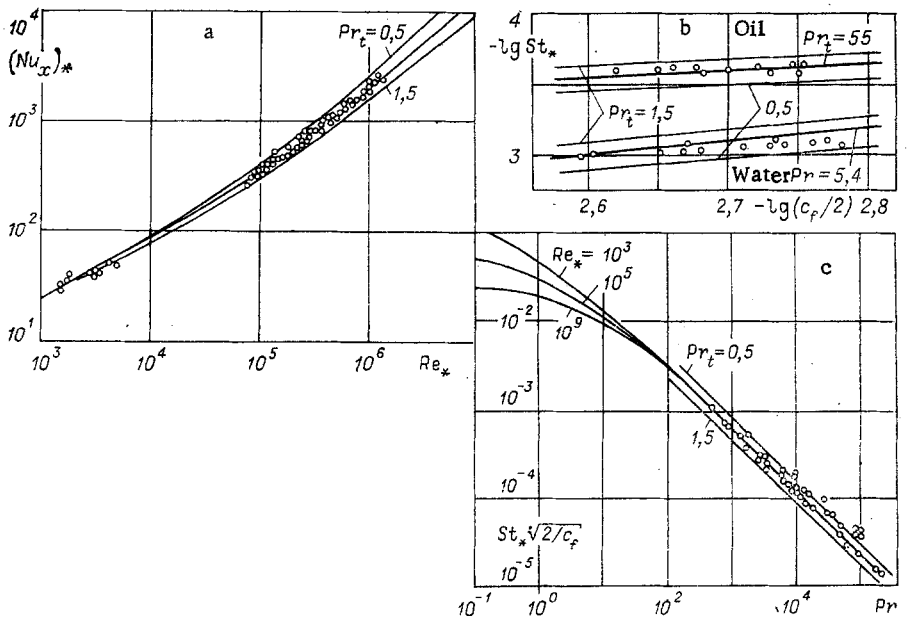

Fig. 4

points belonging to different values of Prandtl and Reynolds numbers are concentrated near the curve and lead to a unique dependence. It is seen from Fig. 3 that the same law is observed even when $Pr_t \neq 1$. For each of the values of turbulent Prandtl numbers the computed points for all values of Pr and Re form a unique curve. It follows from Fig. 3 that for small values of Peclet number the effect of turbulent Prandtl number is negligible. It increases with Pe and becomes a maximum when Pe $\sim 10^{10}$.

A comparison of computed results is given in Fig. 4 for different values of turbulent Prandtl number with experimental data for liquid metals [12] and air [13] (Fig. 4a), water and oil [4] (Fig. 4b), and also with experimental data related to large values of Prandtl number [3] (Fig. 4c). It is seen from Fig. 4 that computed relations obtained at $Pr_t = 0.5$ and 1.5 limit experimental data above and below. Computed curve obtained for $Pr_t = 1$ passes between these curves and agrees much better with experimental data. It is most clearly seen at Prandtl numbers close to unity (Fig. 4a) when the effect of turbulent Prandtl number on computed results is a maximum (see Fig. 2). At small values of Prandtl number the effect of Pr_t is small. In this case all three computed curves practically coincide and agree fairly well with experimental data (Fig. 4a). At large Prandtl numbers there are a larger number of experimental points between computed curves for $Pr_t = 1$ and 0.5 than between computed curves for $Pr_t = 1$ and 1.5 (Fig. 4c). In spite of this, the value of the product $St\sqrt{2/c_f}Pr^{3/4}$ equal to 0.115, which was found in [3] by equating to experimental data and shown in Fig. 4c, agrees best with corresponding computed value 0.113 obtained at $Pr_t = 1$. From this and those given in Fig. 4c it is possible to consider that at large values of Prandtl numbers the best agreement is achieved when turbulent Prandtl number is between 1 and 0.8.

Thus, in computations of heat transfer in boundary layer on a flat plate it is possible to recommend a turbulent Prandtl number equal to unity or somewhat less than unity (at large Prandtl numbers). Stanton numbers obtained in this case agree well with experimental data. Furthermore, computed results at $Pr_t = 1.5$ and 0.5 agree poorly with experimental data except in the region of small Peclet numbers ($Pe < 10^5$) when computed Stanton number weakly depends on the chosen turbulent Prandtl number.

It is necessary to emphasize that these results should be considered only as numerical recommendations keeping in view that in practice, as shown by a number of studies (see, e.g., [3, 14, and 15]), turbulent Prandtl number varies appreciably across the boundary layer, especially near the wall when the nature of this dependence is appreciably affected by Prandtl number [15], thermal boundary condtions [14], and other factors.

LITERATURE CITED

1. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1968).
2. J. Blome and A. A. Vries, "Effect of turbulent Prandtl number," in: Heat and Mass Transfer [in Russian], Vol. 1, Énergiya, Moscow (1968).
3. S. S. Kutateladze, Near-Wall Turbulence [in Russian], Nauka, Novosibirsk (1973).
4. A. A. Zhukauskas and A. A. Shlanchyauskas, Heat Transfer in Turbulent Flow of a Liquid [in Russian], Mintas, Vilnius (1973).
5. A. J. Reynolds, Turbulent Flows in Engineering, Wiley (1974).
6. A. Sh. Dorfman and O. D. Lipovetskaya, "Heat transfer of isothermal flat plate in turbulent flow of a liquid in a wide range of Prandtl and Reynolds numbers," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1976).
7. J. L. Mellor, "Influence of pressure gradient on turbulent flow near a smooth wall," Collection of Translations, Mekhanika, No. 2 (1967).
8. K. K. Fedyaevskii, A. S. Ginevskii, and A. V. Kolesnikov, Computation of Incompressible Turbulent Boundary Layer [in Russian], Sudostroenie, Leningrad (1973).
9. J. L. Mellor and D. M. Gibson, "Equilibrium turbulent boundary layer," Collection of Translations, Mekhanika, No. 2 (1967).
10. A. Sh. Dorfman, "Solution to heat transfer equation for equilibrium boundary layers with arbitrary temperature distribution on the solid surface," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1978).
11. A. Sh. Dorfman, Heat Transfer in Flow Past Nonisothermal Bodies [in Russian], Mashinostroenie, Moscow (1982).
12. E. D. Fedorovich, "Heat transfer of flat plate in incompressible turbulent boundary layer with Pr << 1," Inzh. Fiz. Zh., 2, No. 9 (1959).
13. B. S. Petukhov, A. A. Detlaf, and V. V. Kirillov, "Experimental investigation of heat transfer to a flat plate in subsonic turbulent flow of air," Zh. Tekh. Fiz., 24, No. 10 (1954).
14. A. I. Leont'ev, E. V. Shishov, V. M. Belov, and V. N. Afanas'ev, "Mean and fluctuating characteristics of thermal turbulent boundary layer and heat transfer in a diffuser," in: Heat and Mass Transfer — V [in Russian], Vol. 1, pt. 1, Minsk (1976).
15. A. L. Maksin, B. S. Petukhov, and A. F. Polyakov, "Computation of turbulent heat transfer in stabilized pipe flow," in: Heat and Mass Transfer — V [in Russian], Vol. 1, pt. 1, Minsk (1976).